GOODNESS OF FIT TESTS FOR THE TWO PARAMETER WEIBUL DISTRIBUTION

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Abstract

The estimation of the Weibull density is introduced by a goodness of fit tests. Some numerical results were obtained through a simulation study to obtain the critical values for some well known statistics, beside the power function for these tests.

Key Words: goodness of fit tests, Weibull density, simulation, power function
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1. Introduction

We consider tests of fit based on the empirical distribution function (EDF). The EDF is a step function, calculated from the sample, which estimates the population distribution function. EDF statistics are measures of the discrepancy between the EDF and a given distribution function, and are used for testing the fit of the sample to the distribution; this may be completely specified or may contain parameters which must be estimated from the sample.

Suppose a given random sample of size n is $X_1, X_2, \ldots\ldots, X_n$ and let $X_{(1)} < \ldots\ldots < X_{(n)}$ be the order statistics; suppose further that the distribution of X is $F(x)$ and we assume this distribution to be continuous. The empirical distribution function (EDF) is $F_n(x)$ defined by

$$F_n(x) = \frac{number\ of\ observations \le x}{n}\ ; \quad -\infty < x < \infty. \tag{1}$$

More precisely, the definition is

$$F_n(x) = \begin{cases} 0 & x < X_{(1)} \\ i/n & X_{(i)} < x < X_{(i+1)}, \qquad i = 1,2,\ldots,n-1, \\ 1 & X_{(n)} \le n \end{cases} \tag{2}$$

Thus $F_n(x)$ is a step function, calculated from the data; as x increases it takes a step up of height 1/n as each sample observation is reached. For any x, $F_n(x)$ records the proportion less than or equal to x, while $F(x)$ is the probability of an observation less than or equal to x. We can expect $F_n(x)$ to estimate $F(x)$, and it is in fact a consistent estimator of $F(x)$; as $n \to \infty$, $|F_n(x) - F(x)|$ decreases to zero with probability one.

2-Empricial Distribution Function Statistics

A statistic measuring the difference between $F_n(x)$ and $F(x)$ will be called an Empirical Distribution Function (EDF) statistic. We shall concentrate on several EDF statistics which have attracted most attention. They are based on the vertical differences between $F_n(x)$ and $F(x)$, and are conveniently divided into two classes, "the supremum class" and "the quadratic class".

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The supremum statistics. The first two EDF statistics, D^+ and D^-, are, respectively, the largest vertical difference when $F_n(x)$ is greater than $F(x)$, and the largest vertical difference when $F_n(x)$ is smaller than $F(x)$; formally,

$$D^+ = \sup_x \{F_n(x) - F(x)\} \tag{3}$$

and

$$D^- = \sup_x \{F(x) - F_n(x)\}. \tag{4}$$

The most well-known EDF statistic is D, introduced by Kolmogorov (1933) :

$$D = \sup_x |F_n(x) - F(x)| = \max(D^+, D^-). \tag{5}$$

The quadratic statistics. A second and wide class of measures of discrepancy is given by the Cramér-von Mises family

$$Q = n\int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \psi(x)\, dF(x) \tag{6}$$

where $\psi(x)$ is a suitable function which gives weights to the squared difference $\{F_n(x) - F(x)\}^2$. When $\psi(x) = 1$ the statistic is the Cramér - von Mises statistic, now usually called W^2, and when $\psi(x) = [\{F(x)\}\{1 - F(x)\}]^{-1}$ the statistic is the Anderson-Darling (1954) statistic, called A^2. A modification of W^2 is the Watson (1961) statistic U^2 defined by

$$U^2 = n\int_{-\infty}^{\infty} \left\{ F_n(x) - F(x) - \int_{-\infty}^{\infty} [F_n(x) - F(x)]\, dF(x) \right\}^2 dF(x) \tag{7}$$

From the basic definitions of the supremum statistics and the quadratic statistic given above, suitable computing formulas must be found. This is done by using the Probability Integral Transformation (PIT), $u = F(x)$; when $F(x)$ is the true distribution of X, the new random variable u is uniformly distributed between 0 and 1.

Then u has distribution function $F^*(u) = u,\ 0 \le u \le 1$. Suppose that a sample $X_1, \ldots, X_n$ gives values $u_i = F(x_i),\ i = 1, 2, \ldots, n$, and let $F_n^*(u)$ be the EDF of the values u_i. EDF statistics can now be calculated from a comparison of $F_n^*(u)$ with the uniform distribution for u. It is easily shown that, for values u and x related by $u = F(x)$, the corresponding vertical difference in the EDF diagrams for X and for u are equal; that is,

$$F_n(x) - F(x) = F_n^*(u) - F^*(u) = F_n^*(u) - u\,; \tag{8}$$

Consequently EDF statistics calculated from the EDF of the u_i compared with the Uniform distribution will take the same values as if they were calculated from the EDF of the X_i, compared with $F(x)$. This leads to the following formulas for calculating EDF statistics from the u - values.

The formulas involve the u - values arranged in ascending order $u_{(1)} \le u_{(2)} \le \ldots \le u_{(n)}$. Then, with $\bar{u} = \sum_{i=1}^{n} \frac{u_i}{n}$,

$$D^+ = \max_{1\le i\le n}\left\{ \frac{i}{n} - u_{(i)} \right\};\ D^- = \max_{1\le i\le n}\left\{ u_{(i)} - \frac{(i-1)}{n} \right\};\ D = \max\left(D^+, D^-\right) \qquad (9)$$

$$V = D^+ + D^-\ ,\ \ D_1 = \max_{1\le i\le n}\left| u_i - \frac{i}{n+1} \right|;\ D_{22} = \sum_{i=1}^{n}\left| u_i - \frac{(i+0.5)}{n+1} \right|, D_3 = \sum_{i=1}^{n}\max_{1\le i\le n}\left[\left\{\frac{i}{n} - u_i\right\},\left\{u_i - \frac{(i-1)}{n}\right\}\right]$$

$$W^2 = \sum_{i=1}^{n}\left\{ u_{(i)} - \frac{(2i-1)}{2n} \right\}^2 + \frac{1}{12n};\quad U^2 = W^2 - n(\bar{u} - 0.5)^2$$

$$W_0 = \sum_{i=1}^{n}\left\{ u_i - \frac{(2i-1)}{2n} \right\}^2;\quad W_{11} = \sum_{i=1}^{n}\left\{ u_i - \frac{i}{n+1} \right\}^2,\ W^* = W^2\left(1 + \frac{25}{n}\right)$$

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln u_{(i)} + \ln\left\{1 - u_{(n+1-i)}\right\}\right].$$

Another formula for A^2 is $A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}\left[(2i-1)\ln u_{(i)} + (2n+1-2i)\ln\left\{1 - u_{(i)}\right\}\right]$

$$A_{21} = -n - \frac{n}{(n+1)^2}\left[\sum_{i=1}^{n}\{(2i-1)\ln u_i + (2i+1)\ln(1 - u_{n+1-i})\} - \{(2n+1)\ln u_n - \ln(1 - u_n)\}\right]$$

$$A_{12} = -(n+1) - \frac{1}{n+1}\sum_{i=1}^{n} i\left\{\ln u_i + \ln\left(1 - u_{n+1-i}\right)\right\}$$

$$A^* = \left(1 + \frac{4.2}{n} - \frac{43}{n^2}\right)A^2 \qquad (10)$$

3-Goodness Of Fit Tests Based On The EDF Tests

The general test of fit is a test of the null hypothesis

H_0 : a random sample of n X - values comes from $F(x;\theta)$

where $F(x;\theta)$ is a continuous distribution and θ is a vector of parameters. When θ is fully specified i.e. the parameters are known. Then $u_{(i)} = F\left(x_{(i)};\theta\right)$ gives a set $u_{(i)}$ which, on H_0, are ordered uniforms and equations (9) are used to give EDF statistics. On the other hand, $F(x;\theta)$ may be defined only as a member of a family of distributions, but all or part of the vector θ may be known.

When θ is known, distribution theory of EDF statistics is well-developed, even for finite samples, and tables are available for some time. When θ contains one or more unknown parameters, these parameters may be replaced by estimates, to give $\hat{\theta}$ as the estimate of θ. Then formulas (9) may still be used to calculate EDF statistics, with $u_{(i)} = F\left(x_{(i)};\hat{\theta}\right)$. However, even when H_0 is true, the $u_{(i)}$ will now not be an ordered uniform sample, and the distributions of EDF statistics will be very different from those when θ is known, they will depend on the distribution tested, the parameters estimated, and the method of estimation, as well as on the sample size. New points should then be used for the appropriate test, even for large samples, otherwise a serious error in significance level will result.

3.1-Unknown location and scale parameters

When the unknown components of θ are location or scale parameters, and if these are estimated by appropriate method, the distributions of EDF statistics will not depend on the true values of the unknown parameters. Thus percentage points for EDF for such distributions, depend only on the family tested and on the sample size n. Nevertheless, the exact distributions of EDF statistics are very difficult to find and except for the exponential distribution, Monte Carlo studies have been extensively used to find points for finite n. Fortunately, for the quadratic statistics W^2, U^2 and A^2, asymptotic theory is available; furthermore, the percentage points of these statistics for finite n converge rapidly to the asymptotic points. For the statistics D^+, D^-, D and V, there is no asymptotic theory and even asymptotic points must be estimated.

For the tests corresponding to many distributional families, Stephens (1970,1974, 1977,1979) has given modification of the test statistics; if the statistic is, say, T the modification is a function of n and T which is then referred to the asymptotic points of T or of $T\sqrt{n}$. Asymptotic theory depends on using asymptotically efficient estimators for the estimates of unknown components of θ; the asymptotic points given will then be valid for any such estimators. Points for finite n will depend on which estimators are used; usually these are maximum likelihood estimators.

3.2-Unknown shape parameters

When unknown parameters are not location or scale parameters, for example when the shape parameter of a Gamma or a Weibull distribution is unknown, null distribution theory, even asymptotic, when the parameters are estimated, will depend on the true values of these parameters. However, if this dependence is very slight, a set tables, to be used with the estimated value of the shape parameter, can still be valuable.

4-Miscellaneous Topics on EDF Tests

4.1-Power of EDF statistics when parameters are estimated

Different statistics were found to detect different types of departure from uniformity. When unknown parameters are estimated from the same sample as is used for the goodness of fit test, the differences in the powers of the statistics tend to be smaller. It appears that fitting the parameter or parameters makes it possible to adjust the tested distribution to the sample in such a way that the statistics can detect a departure from the null distribution with roughly the same efficiency; nevertheless, A^2 tends to lead the others, probably because it is effective at detecting departures at the tails.

Some asymptotic theory is available to examine power, at least for quadratic statistics. Dubrin and Knott (1972) demonstrated a method by which asymptotic power results could be obtained, and applied it to test for the normal distribution with mean 0 and variance 1, that is, when the parameters are known, against normal alternatives with a shift in mean or a shift in variance. Stephens (1974a) extended the results to shifts in both mean and variance. The technique rests on a partition of the appropriate statistics into components. Dubrin, Knott and Taylor (1975) showed how the decomposition into components could be done also for the test for normality with mean and variance unknown, or for the exponential test with scale parameter unknown and used their method to discuss the asymptotic power of the components. Stephens (1976) followed the method and applied it to tests for the statistics W^2, U^2

and A^2 for these situations. The overall result when tests for normality or exponentially are made with unknown parameters, is that A^2 is slightly better than W^2 for the alternatives discussed, with U^2 not far behind W^2.

The superiority of A^2 has also been documented by various power studies based on Monte Carlo sampling. Some of these, in comparisons of tests of uniformity and and normality, are by Stephens (1974b). These power studies also included the statistics D^+, D^-, D and V.

The most famous statistic, the Kolmogorov-Smirnov D, tends to be weak in power. Statistics D^+ and D^-, on the other hand, often have good power but each one against only certain classes of alternatives. In some applications the alternatives of interest may be clearly identified, and then it will be possible to identify which statistic to use. However, D^+ and D^- will be biased when used against the wrong alternatives, so these statistics must be used with caution.

4.2-The effect on power of knowing certain parameters

It is usually assumed in statistical testing that the more the knowledge the better. However, the tests are (ideally) intended as tests for distributional form, not as tests for parameters values, and some knowledge of parameters may not be very important in assessing distributional form. For example, it may be unhelpful to know, and to use, the mean of the true distribution, when this is not the one tested. Stephens (1974b) and Dyer (1974) have noted these effects in tests for normality; being given means and variances changes the test from the case when parameters are unknown to the case when the parameters are known, with a consequent loss of power. On the other hand, Spinelli and Stephens (1983) have shown that in tests for exponentially it is better to use the value of the origin, when this is known, than to estimate it. Further work is still needed on what parametric information is useful and what is not.

4.3-Use of sufficient statistics

Some other interesting method have been proposed to deal with unknown parameters. When sufficient statistics are available for θ, Srinivasan (1970,1971) has suggested using the Kolmogorov statistic D calculated from a comparison of $F_n(x)$ with the estimate $\tilde{F}(x;\hat{\theta})$ obtained by applying the Rao-Blackwell theorem to $F(x;\hat{\theta})$, where $\hat{\theta}$ is, say, the maximum likelihood estimator of θ. The resulting tests are asymptotically equivalent to the tests using $F(x;\hat{\theta})$ itself (Moore (1973)) and can be expected to have similar properties for finite n. The method will usually lead to complicated calculations, and has been developed only for tests for normality (Srinivasan(1970) and Kotz (1973)) and tests for exponentially.

5-EDF Tests for The Weibull Distribution

In this section we are concerned with goodness-of-fit tests for the Weibull distribution which is given by

$$f(x;\alpha,\beta)=\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\},\quad x>0 \qquad (11)$$

and whose distribution function takes the form

$$F(x;\alpha,\beta)=1-\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x>0 \tag{12}$$

when the parameters α and β are unknown and are estimated from a random sample. Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample from the Weibull distribution with parameters α and β. It is well known that the maximum likelihood estimators of α and β are obtained by solving

$$\frac{n}{\hat{\beta}}-\frac{n\sum_{i=1}^{n} x_i^{\hat{\beta}} \ln x_i}{\sum_{i=1}^{n} x_i^{\hat{\beta}}}+\sum_{i=1}^{n} \ln x_i = 0 \tag{13}$$

and

$$\hat{\alpha}=\left(\frac{\sum_{i=1}^{n} x_i^{\hat{\beta}}}{n}\right)^{1/\hat{\beta}}. \tag{14}$$

Let $y_i = \ln x_i$, $i = 1,2,\ldots\ldots,n$ and $u_{(i)} = F\left(x_{(i)};\hat{\alpha},\hat{\beta}\right)$ where $x_{(i)}$ is the i th order statistic.

5.1-The test statistic

The null hypothesis is

H_0: the random sample $x_1, x_2, \ldots\ldots, x_n$ comes from the Weibull distribution $f\left(x;\alpha,\beta\right)$ with the unknown parameters α and β.

We will concentrate on the following main tests for goodness of fit for the Weibull distribution.

1-Modified Kolmogorov-Smirnov statistic D :

$$D=\sup\left|F\left(x;\hat{\alpha},\hat{\beta}\right)-F_n\left(x\right)\right|=\sup\left|\left(1-\exp\left[-\left(\frac{x}{\hat{\alpha}}\right)^{\hat{\beta}}\right]\right)-F_n\left(x\right)\right|,$$

where $F_n(x)=\dfrac{(\# X_i \le x)}{n}$ is the empirical distribution function of the sample.

This is equivalent to $D=\max\left(D^+, D^-\right)$ Where

$$D^+ = \max_{1\le i\le n}\left\{\frac{i}{n}-u_{(i)}\right\}, \; D^- = \max_{1\le i\le n}\left\{u_{(i)}-\frac{(i-1)}{n}\right\} \tag{15}$$

2-Modified Cramér-von Mises statistic W^2 :

$$W^2 = n\int_{-\infty}^{\infty}\left\{F_n(x)-F(x)\right\}^2 \, dF(x)$$

where $F_n(x)$ is defined in equation (1). Put $u = F(x)$ where

$$U_n(n)=\begin{cases}0 & u<U_{(1)} \\ i/n & U_{(i)}<u<U_{(i+1)}, \qquad i=1,2,\ldots,n-1 \\ 1 & U_{(n)}\le n\end{cases}$$

$u_{(0)}=0$ and $u_{(n+1)}=1$. Then

$$W^2=n\int_0^1\{U_n(n)-u\}^2\,du=n\sum_{i=0}^{n}\int_{u_{(i)}}^{u_{(i+1)}}\left[\frac{i}{n}-u\right]^2 du$$

$$=\frac{n}{3}\sum_{i=1}^{n}\left[\left(u_{(i+1)}-\frac{i}{n}\right)^3-\left(u_{(i)}-\frac{i}{n}\right)^3\right]$$

$$=\sum_{i=1}^{n}\left\{u_{(i)}-\frac{(2i-1)}{2n}\right\}^2+\frac{1}{12n}. \qquad (16)$$

3-Modified Anderson-Darling statistic A^2 :

$$A^2=n\int_{-\infty}^{\infty}\{F_n(x)-F(x)\}^2\,\frac{1}{F(x)(1-F(x))}\,dF(x)$$

$$=n\int_0^1\{U_n(n)-u\}^2\,\frac{1}{u(1-u)}\,du$$

$$=-n-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln u_{(i)}+\ln\{1-u_{(n+1-i)}\}\right]. \qquad (17)$$

The results of David and Johnson (1948) imply that the distributions of D, W^2 and A^2 can be conducted without loss of generality by fixing $\alpha=\beta=1$. (See Tho-man, Bain and Antle (1969) for results may be used to arrive at the same conclusion, specific to the Weibull distribution).

5.2-Simulation study

The idea of this simulation study is two things; first: to show the goodness of fit test for the Weibull distribution by using the test statistics D, W^2 and A^2 for the sample of size n and second: to compare between the power functions of test statistics D, W^2 and A^2 for several distributions.

The following table represents the critical values of D, W^2 and A^2 when n=10(5)40 at α=.20,.15,.10,.05,.01 respectively.

We notice from the results shown in Table 5.1 that the critical values of D decreases while n increases and they increase when α decreases. However, the critical values of W^2 and A^2 are varies for different values of n while they increase when α decreases.

Table 5.2 represents the power functions of D, W^2 and A^2 for Weibull, pareto, and chi - square distributions when n=10, 15, 20, 30 and α=.20,.15, .10, .05, .01 where we will compare between the distributions using the critical values of D, W^2 and A^2 from Table 5.1.

We notice from table 5.2 that the power functions of D, W^2 and A^2 for Weibull and chi - square distributions are similar to each other and they converge to the values of α for different values of n while the power functions of D, W^2 and A^2 for the pareto distribution diverges from the values of α when n increases. This shows that the power functions of D, W^2 and A^2 for Weibull and chi-square distributions are better than the power functions of D, W^2 and A^2 for the pareto distribution.

Appendix

Table 5.1 The critical values of D, W^2 and A^2 when n=10(5)40.

n	statistic \ α	.20	.15	.10	.05	.01
10	D	0.2168	0.2269	0.2396	0.2594	0.2973
	W^2	0.0789	0.0874	0.0988	0.1181	0.1627
	A^2	0.5003	0.5439	0.6091	0.7156	0.9734
15	D	0.1802	0.1883	0.1992	0.2162	0.2503
	W^2	0.0797	0.0880	0.0996	0.1201	0.1664
	A^2	0.5042	0.5522	0.6189	0.7316	0.9947
20	D	0.1579	0.1652	0.1748	0.1886	0.2188
	W^2	0.0799	0.0884	0.1006	0.1208	0.1680
	A^2	0.5070	0.5554	0.6235	0.7371	0.9970
25	D	0.1417	0.1482	0.1566	0.1702	0.1947
	W^2	0.0791	0.0877	0.0993	0.1198	0.1652
	A^2	0.5041	0.5523	0.6189	0.7294	0.9832
30	D	0.1298	0.1359	0.1435	0.1553	0.1795
	W^2	0.0790	0.0879	0.0997	0.1195	0.1651
	A^2	0.5063	0.5539	0.6219	0.7333	0.9922
35	D	0.1208	0.1264	0.1336	0.1448	0.1679
	W^2	0.0790	0.0881	0.1003	0.1209	0.1675
	A^2	0.5059	0.5564	0.6218	0.7348	0.9933
40	D	0.1130	0.1181	0.1251	0.1356	0.1581
	W^2	0.0791	0.0877	0.0994	0.1202	0.1697
	A^2	0.5049	0.5554	0.6235	0.7379	1.0074

Table 5.2. The power functions of D, W^2 and A^2 for different distributions when n=10,15,20,30.

n	α \ p.f.	Weibull $P(D)$	Weibull $P(W^2)$	Weibull $P(A^2)$	Pareto $P(D)$	Pareto $P(W^2)$	Pareto $P(A^2)$	Chi square $P(D)$	Chi square $P(W^2)$	Chi square $P(A^2)$
10	.01	0.7290	0.8095	0.8185	0.2305	0.3765	0.3480	0.0130	0.0115	0.0100
	.05	0.0455	0.0490	0.0480	0.4415	0.5875	0.5920	0.0540	0.0570	0.0540
	.10	0.0960	0.0970	0.0925	0.5780	0.6965	0.6945	0.1065	0.1160	0.1075
	.15	0.1395	0.1420	0.1380	0.6665	0.7670	0.7700	0.1630	0.1710	0.1640
	.20	0.1840	0.1850	0.1845	0.7290	0.8095	0.8185	0.2165	0.2335	0.2150
15	.01	0.0090	0.0110	0.0105	0.4485	0.6485	0.6590	0.0070	0.0100	0.0055
	.05	0.0540	0.0510	0.0545	0.7030	0.8135	0.8295	0.0490	0.0525	0.0470
	.10	0.0925	0.0925	0.0970	0.8085	0.8885	0.9035	0.0970	0.1100	0.1035
	.15	0.1435	0.1440	0.1420	0.8605	0.9210	0.9350	0.1610	0.1620	0.1555
	.20	0.1950	0.1995	0.1875	0.8605	0.9210	0.9350	0.2090	0.2170	0.2040
20	.01	0.0090	0.0120	0.0135	0.6915	0.8290	0.8565	0.0110	0.0135	0.0125
	.05	0.0595	0.0590	0.0620	0.8765	0.9310	0.9435	0.0670	0.0705	0.0635
	.10	0.1105	0.1140	0.1160	0.9270	0.9575	0.9690	0.1205	0.1275	0.1260
	.15	0.1655	0.1695	0.1615	0.9470	0.9690	0.9800	0.1765	0.1905	0.1855
	.20	0.2160	0.2225	0.2125	0.9605	0.9795	0.9910	0.2305	0.2315	0.2380
30	.01	0.0120	0.0100	0.0080	0.9345	0.9670	0.9795	0.0150	0.0225	0.0180
	.05	0.0500	0.0555	0.0620	0.9845	0.9935	0.9980	0.0765	0.0870	0.0865
	.10	0.0995	0.1045	0.1105	0.9950	0.9985	1.0000	0.1310	0.1540	0.1520
	.15	0.1495	0.1560	0.1605	0.9980	0.9995	1.0000	0.1895	0.2065	0.2030
	.20	0.2070	0.2130	0.2140	0.9980	1.0000	1.0000	0.2400	0.2595	0.2515

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